Proceedings of ICONE-23
23rd International Conference on Nuclear Engineering
May 17-21, 2015, Chiba, Japan

ICONE23-1692

3D PWR POWER DISTRIBUTION ON-LINE MONITORING BASED ON HARMONICS EXPANSION

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Keywords: On-line monitoring, Harmonics expansion method, Full core harmonics calculation, Nonlinear iteration semi-analytic nodal method, DayaBay NPP detector measurements

ABSTRACT

Harmonics expansion method is employed to obtain the spatially continuous 3D on-line power distribution based on the dispersed and limited detector measurements. Power distribution is expanded by harmonics which are high-order eigenfunctions of neutron diffusion equation. The expansion coefficients are determined by using in-core detector measurements. Krylov sub-space method is employed to obtain those harmonics, while least square principle is chosen for expansion coefficients calculation. Moreover, instead of the original quarter core finite differencing method, a whole core nonlinear iteration semi-analytic nodal method is used for harmonics calculation. It has been found that the nodal harmonics calculation runs about 100 times faster than the finite differencing one without any loss in accuracy. Based on these models, an on-line monitoring system named NECP-ONION has been developed. Real detector measurements from Unit 1 reactor of DayaBay NPP, a typical PWR reactor in China, are used for code verification and validation. Numerical results show that the root-mean-square errors of assembly averaged powers are less than 1.8% for different burnup steps during the entire cycle. In addition, it has been observed that the assembly power monitoring error can still be driven down to less than 2% even if only 60% of measurements are available.

1. INTRODUCTION

On-line monitoring of 3D power distribution is pretty important for a nuclear reactor, simply due to two reasons. Firstly, 3D power distribution is one of the most direct responses for reactor core safety but it changes complexly and continuously. Secondly, power distribution on-line monitoring can be employed to reduce the over-conservative operating principles (Luo, 2000). Usually, 3D power distribution on-line monitoring is based upon a number of neutron detector measurements at strategically selected locations in or out of the core. Because the number of detectors is limited and their locations are discrete. The most important target for 3D power distribution on-line monitoring is to estimate the spatial continuous distribution from the discrete measurements in real-time.

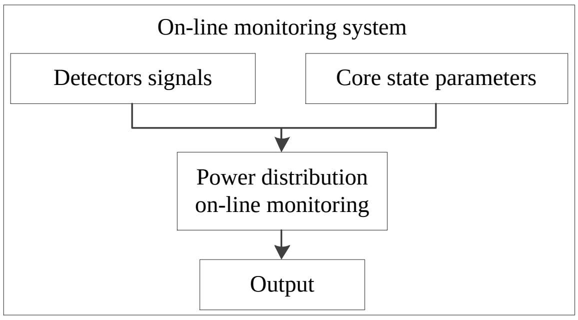

Fig. 1 Flow chart of a typical power distribution on-line monitoring system

As shown in Fig. 1, an integrated on-line monitoring system usually contains four steps: (1) obtain detector measurements; (2) obtain core state parameters such as control rod positions, boron concentration and fuel burnup; (3) power distribution on-line monitoring based on core state parameters and detector measurements; (4) results output.

According to their locations in or out of the core, neutron detectors can be classified into in-core and ex-core detectors.

Two types of in-core neutron detectors have been widely used including movable and fixed detectors. Among those, fixed in-core detectors is the nature choice for power distribution on-line monitoring considering the number of them in a core and their measuring period (Wang, 2011). According to detector theory, self-power neutron detector (SNPD) is selected as the fixed in-core detector (Terney, 1983). The detector signals are electric signals but measurements that we need is power or flux measurements in power distribution on-line monitoring.

Except detector measurements, power distribution on-line monitoring system also requires core state parameters including control rod positions, boron concentration and fuel burnup.

As mentioned before, power distribution on-line monitoring system transforms the discrete measurements to spatially continuous 3D distribution in real-time. To combine the pre-calculated power distribution with on-line measurements, four typical methods are usually adopted. They are harmonics expansion method (Li, 1997), simulation and correction method (Beard, 1988), internal boundary condition method (Chan, 1990), and least squares method (Lee, 2003). (1) Harmonics expansion method, also called modal expansion method, time synthesis approximation or flux synthesis method, is the oldest one developed by AECL (Bonalumi, 1985). Its initial purpose was to calibrate zone-control-detectors by providing average zone fluxes for on-line spatial power control. Now it's still in use for on-line flux mapping CANDU6. In this method, reactor core power distribution is expanded by harmonics, which are high-order eigenfunctions of neutron diffusion equation. Detector measurements are used to determine the expansion coefficients. For CANDU reactors, this method has been verified by industry. Accordingly, if the expanded function is the error of calculated distribution and the basis functions are error shape basis functions, then it becomes error shape synthesis method (Hong, 2004). (2) Simulation and correction method is another method used by industries, such as BEACON (Beard, 1988) by Westinghouse. According to the core condition parameters, neutron diffusion equation is solved to obtain the theoretical core power distribution and the theoretical detector measurements, called predicted or calculated values. What follows is fitting the ratios of detector measured values over theoretical values to obtain a spatial continuous proportion function defined in the entire core. Finally, the monitoring result of power distribution can be obtained by multiplying the proportion function to the predicted power distribution. (3) In the internal boundary condition method, proposed by Chan (1990), neutron diffusion equation and detector measurements are combined by solving neutron diffusion equation directly using the measurements as its internal boundary conditions. It was improved by introducing Kalman filtering technique to reduce calculation and measurement errors (Jeong, 2000). (4) Similarly, in the least squares method (Hong 2005), neutron diffusion function is coupled with detector response equation representing the relationship between neutron flux and detector signals on the least squares principle.

In this paper, there are two assumptions. Firstly, accurate power can be provided by detectors, which excludes the consideration of the relationship between detector signal and measurement. Secondly, all of the core state parameters are exact.

An on-line monitoring system named NECP-ONION (On-line monitoring) has been developed based on harmonics expansion method in Nuclear Engineering Computational Physics (NECP) lab. We chose the harmonics expansion method simply because of its mature theory and industry verification. The target core for verification and validation is DayaBay nuclear power plant reactor core, a typical PWR operating in China. The referred data is obtained from the real detector measurements and commercial code afforded by the plant.

The rest of this paper is organized as follows. Section 2 introduces monitoring method, harmonics expansion method for simply and discusses several questions that have to be considered in this method. Monitoring results and detector effect analyses are performed in Section 3, while conclusions are summarized in Section 4.

2. HARMONICS EXPANSION METHOD

For description convenient, harmonics definition is introduced first. Neutron diffusion equation can be written into operator form as below:

$$M\Phi = \frac{1}{k}F\Phi \qquad \text{Eq. 1}$$

where M and F are neutron disappear and generation operators (cm^{-1}), respectively, Φ and k refer to neutron flux ($cm^{-2}s^{-1}$) and effective multiplication factor of the system.

Define operator $A=M^{-1}F$, Eq. 1 can be written as following:

$$A\Phi = k\Phi \qquad \text{Eq. 2}$$

Eq. 2 is an eigenvalue equation and has serial eigenvalues $\{k_1, k_2, \dots, k_N\}$ and the corresponding eigenfunctions $\{\Phi_1, \Phi_2, \dots, \Phi_N\}$. These eigenvectors are usually called harmonics. Especially, the first order harmonic Φ_1, also called as basic or fundamental mode, is neutron flux distribution and k_1 is the effective multiplication factor.

Harmonics compose a set of complete orthogonal basis functions for its completeness and orthogonality (Li, 1996). Reactor power distribution can be expanded by harmonics:

$$P(r) = \sum_{n=1}^{N} a_n \Phi_n(r) \qquad \text{Eq. 3}$$

where $P(r)$ is the expanded power distribution, r is the spatial variable, N is the number of harmonics and a_n is the nth-order expansion coefficient.

As the harmonics are calculated one step before the on-line monitoring, the expansion coefficients are the only unknowns. Then fixed in-core detector measurements are used to obtain the power distribution in real-time.

$$R(r_d) \propto P(r_d) = \sum_{n=1}^{N} a_n \Phi_n(r_d) \qquad \text{Eq. 4}$$

where $R(r_d)$ is the detector measurement located at r_d.

If the number of detectors is N_d, equations can be obtained as following:

$$\begin{cases} R(r_1) \propto \sum_{n=1}^{N} a_n \Phi_n(r_1) \\ R(r_2) \propto \sum_{n=1}^{N} a_n \Phi_n(r_2) \\ \cdots \\ R(r_{N_d}) \propto \sum_{n=1}^{N} a_n \Phi_n(r_{N_d}) \end{cases} \quad \text{Eq. 5}$$

Then expansion coefficients can be calculated from Eq. 5 through least square principle when $N_d \geqslant N$, and power distribution is reconstructed according to Eq. 3.

As mentioned in Section 1, it is assumed that detector measurements can be obtained accurately. Thus, the treatment about detectors will not be discussed here.

2.1 Harmonics calculation

Power iteration method (Belchior, 1991) and Krylov subspace method (Warsa, 2004) are both effective for harmonics calculation. Krylov subspace method is efficient in solving large matrix eigenvalue problems and widely used recent years for its high calculation speed (Verdu, 1999). Thus, we choose the Krylov subspace method by using an open source mathematics tool named "ARPACK" (Wright, 2002). In addition, two numerical methods are discussed in this paper. One is fine mesh finite difference method (FMFDM) and while the other is nonlinear iteration semi-analytic nodal method (NLSANM) (Liao, 2002).

For NLSANM, computational time for harmonics is pretty short make on-line harmonics calculation possible. For FMFDM, it would be too expensive, which forces the pre-calculated harmonics library (Wang, 2011).

Traditionally, calculation in quarter-core based on symmetry for solving neutron diffusion equation is a tacit way to save computer time and storage. However, it is not suitable for harmonics calculation due to their asymmetry. In addition, power distribution expansion theory requires the same boundary condition for both the expanded power distribution and the harmonics making the quarter core harmonics can only be used to expand the quarter core power distribution.

2.2 Expansion order determination

Theoretically, harmonics expansion order N in Eq. 3 should be infinite. But practically it is selected as a finite number, which brings up the question of how to determine this number N? The finite expansion order N is determined by comparing the accuracies with different numbers of harmonics (Mishra, 2012). Take CANDU as an example, N=15 would be good enough. Fig. 2 shows the accuracy of on-line monitoring power distribution varies with the number of harmonics for a typical PWR and indicates 15 would be acceptable. Two reasons together result in the steep drop of relative error from N=14 to N=15. The first one is that the drop of root-mean-square error is leaded by the drop of maximum error; the second one is that the fifteenth harmonic shape is just similar with the monitoring power distribution error shape and its attendance makes up the difference between monitoring power distribution and the reference one. But it has to be noticed that this is problem dependent. Different problem has different error shape and needs different order of harmonics to make it up.

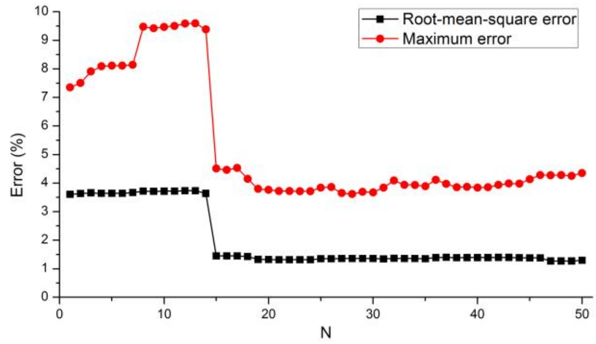

Fig. 2 Errors of monitoring power distributions by different expansion order

2.3 Detector failure

In this paper, the treatment for failure detectors is to directly ignore them. Then Eq. 5 becomes:

$$\begin{cases} R(r_1) \propto \sum_{n=1}^{N} a_n \Phi_n(r_1) \\ \cdots \\ R(r_{N_f-1}) \propto \sum_{n=1}^{N} a_n \Phi_n(r_{N_f-1}) \\ R(r_{N_f+1}) \propto \sum_{n=1}^{N} a_n \Phi_n(r_{N_f+1}) \\ \cdots \\ R(r_{N_d}) \propto \sum_{n=1}^{N} a_n \Phi_n(r_{N_d}) \end{cases} \quad \text{Eq. 6}$$

where N_f is the failure detector number.

3. NUMERICAL RESULTS AND DISCUSSION

Based on the above theory, an on-line monitoring system named NECP-ONION has been developed in our NECP lab. It calculates harmonics, monitors power distribution on-line and handle the detector failure. Therefore, numerical results in this paper consists two parts, respectively for harmonics calculation and power distribution monitoring. All calculations were performed on a PC with Inter Core i7-3770 CPU and 3.47 GB RAM.

3.1 Harmonics calculation

Two benchmarks, IAEA-3D and LMW-3D, were carried out to validate the harmonics calculation. Table 1 shows the first-order eigenvalues calculated based on the two methods, FMFDM and NLSANM, and compared with the references. The first ten eigenvalues are presented in Fig.3. DayaBay nuclear power plant reactor core 13th cycle first twenty orders eigenvalues are also presented when the burnup is 0 MWd/tU in Fig.4. It is shown that both FMFDM and NLSANM have good accuracy.

Table 1 First eigenvalues comparison

Benchmark	Reference*	FMFDM (Error/pcm)	NLSANM (Error/pcm)
IAEA-3D	1.02903	1.02884(19)	1.02911(8)
LMW-3D	0.99966	0.99964(2)	0.99978(12)

* Reference is from reference (Liao, 2002)

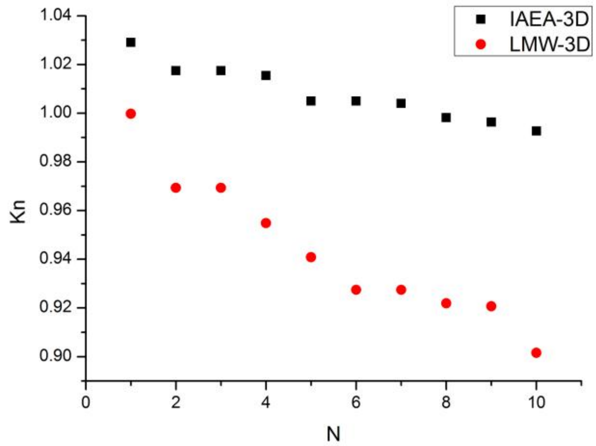

Fig. 3 First ten orders eigenvalues of IAEA-3D and LMW-3D benchmark calculated by NLSANM

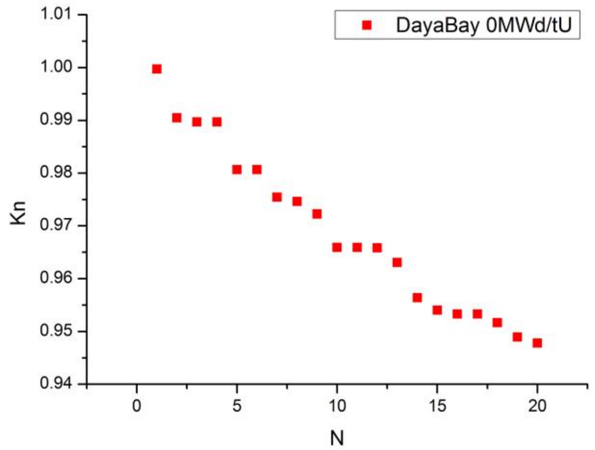

Fig. 4 First twenty orders eigenvalues of DayaBay calculated by NLSANM

To obtain the first twenty harmonics, computational times of the two methods are compared in Table 2. It shows that NLSANM runs about 100 times faster than FMFDM. Thanks to its high efficiency, nodal harmonics on-line calculation becomes possible.

Table 2 computational time comparison for FMFDM and NLSANM

Problem	Mesh(x*y*z)		Computer time(s)	
	FMFDM	NLSANM	FMFDM	NLSANM
IAEA-3D	119*119*95	17*17*19	3004	33
LMW-3D	110*110*50	11*11*10	680	5
DayaBay 0MWd/tU	136*136*90	17*17*18	3849	25

3.2 Power distribution on-line monitoring

For verification and validation of on-line monitoring system, parameters of Unit 1 reactor of DayaBay NPP 13th cycle have been used. Comparisons have been carried out, including FMFDM code and nuclear plant operating data. In addition, part detectors failure is also discussed. As NLSANM has much less computational time than FMFDM, all harmonics from now on were calculated by NLSANM.

3.2.1 Problem description

Reactor core of DayaBay NPP is a typical PWR with key parameters summarized in Table 3. The core consists of 157 assemblies. Each assembly incorporates 264 fuel rods, 24 guide thimbles and 1 instrumentation tube. Gadolinium poison is used in the core to control the reactivity and the power distribution.

Table 3 Summary of key model parameters for test core

Parameter	Value	Parameter	Value
Core power	2895MW th	Operating pressure	15.5M Pa
No. fuel assemblies	157	Pin lattice configuration	17x17
No. fuel rods in one assembly	264	Active fuel length	365.76 cm
Burnable poison material	Gd_2O_3		

The 13th cycle reloading scheme is shown in Fig. 5 with a low leakage loading strategy. 72 fresh fuel assemblies with an enrichment of 4.45% are loaded. And there are also assemblies from 11th cycle and 12th cycle loaded as shown in Fig. 5.

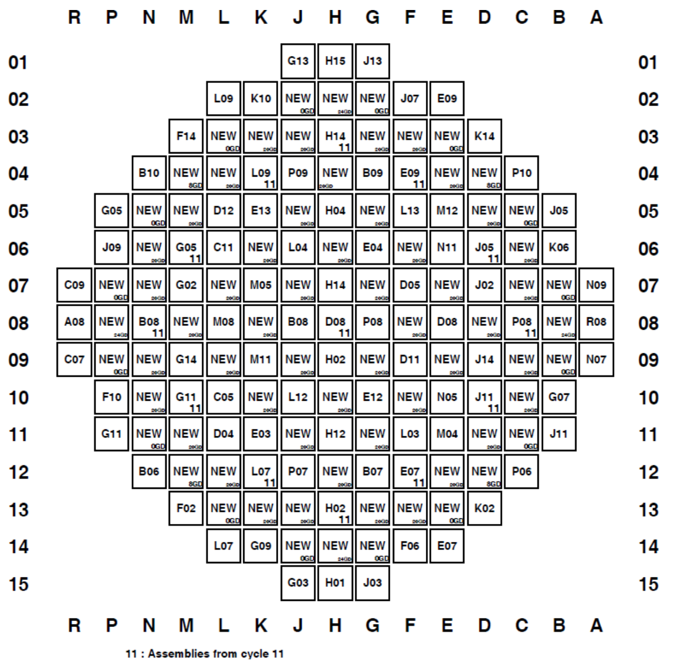

Fig. 5 Cycle 13 reshuffling pattern

There are two categories of control rod clusters: control rod banks and shutdown rod banks. Control rod banks is divided into two types: power compensation banks (G1, G2, N1 and

N2), used to control the core reactivity changes associated with step load changes; temperature regulation bank (R), used to control the core average temperature. Shutdown rod banks (SA, SB, SC and SD) drop into the core simultaneously with control rod banks in case of reactor trip to ensure the required negative reactivity.

Practically, there is no in-core fixed detector installed in the DayaBay reactor core. To verify the accuracy of NECP-ONION, movable detector measurements are used to simulate the fixed in-core detectors. It is assumed that the "fixed detectors" are located in the same tubes with the movable ones in radial and there are five of them in each tube in axial as shown in Fig. 6. These "fixed detectors" are named according to their assembly locations such as detector "A09". Each of these pseudo "fixed detector" is 32.085cm long. Their measurements are calculated according to the movable detector measurements happened in their axial height with the volume weighting principle. In addition, this movable detector system also can provide core power distribution which be used in Section 3.2.3 as reference. Both these distribution and movable detector measurements are afforded by the DayaBay NPP.

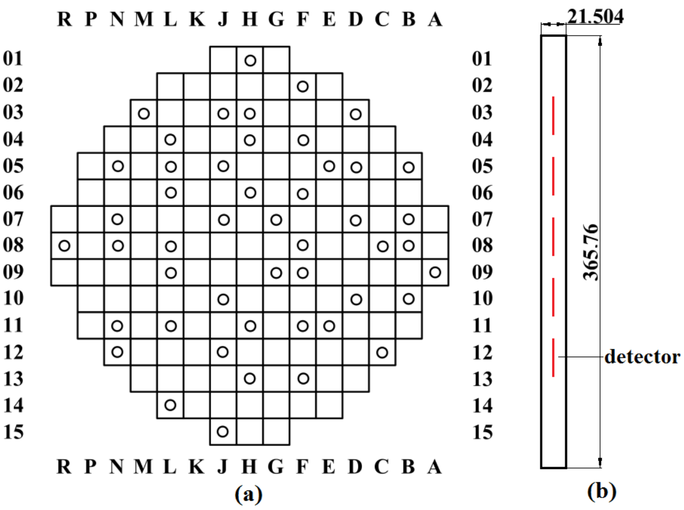

Fig. 6 Assembly locations with detectors

3.2.2 Comparison with theoretical calculation

If the measurements from a plant are absent, several assumed operating states should be carried out to recommend the on-line monitoring system's adaptability. Then the FMFDM code was employed. Fig. 7 shows the flow chart of the comparison with theoretical calculation. It contains four steps: (1) calculate theoretical power distribution by the FMFDM code at the monitored core condition; (2) simulate detector measurements based on theoretical power distribution; (3) calculate on-line monitored power distribution; (4) compare monitored power distribution with the theoretical one and output.

Reactor core operating states were considered as changes of burnup and boron concentration. Monitoring cases and meshes are listed in Table 4 and Table 5. These cases' root-mean-square errors and maximum errors of radial average power distribution are figured out in Fig.8. It shows that the monitored power distributions agree well with the reference. The root-mean-square errors are less than 0.2% and the maximum errors are less than 1% for all cases.

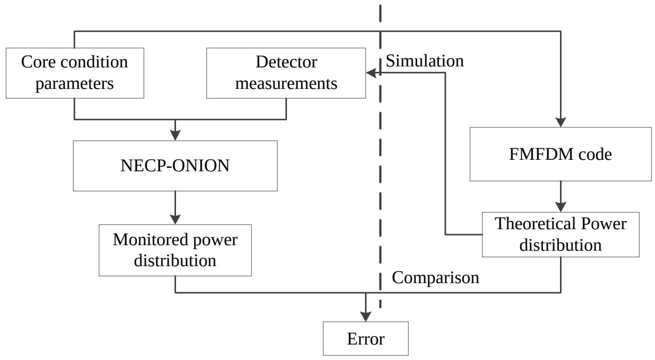

Fig. 7 Flow chart of the comparison between on-line monitoring system and theoretical calculation

Table 4 Calculation cases

State	Case
Burnup(MWd/tU)	0 150 500 1000 2000 5000 10000 15000 20000
Boron concentration(ppm)	Critical boron concentration in each burnup
Control Rod	ARO

Table 5 Calculation meshes

	Reference(FMFDM)	NLSANM
Meshes(x*y*z)	340*340*90	34*34*18

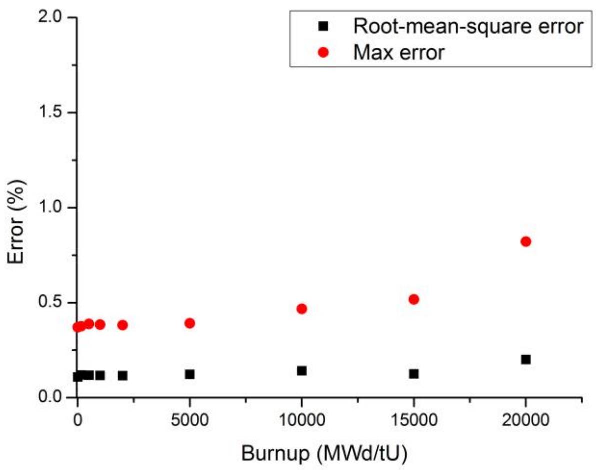

Fig. 8 Root-mean-square errors and maximum errors of radial average power distribution for the operating state

3.2.3 Comparison with NPP on-line monitoring results

Detector measurements and monitored power distributions can be obtained from nuclear plant. With the same detector measurements as input, NECP-ONION is compared with these monitored power distributions. Monitoring was carried out from the beginning of cycle (BOC) to the end of cycle (EOC). Fig. 9 shows the root-mean-square errors (RMS) and maximum errors of the radial average power distributions. Two of these states are selected to compare their radial average power distributions errors of assemblies. One is close to BOC, while the other is close to EOC.

As shown in Fig.9, all the RMS errors are less than 1.8%, while all the maximum errors are less than 5.0%. Fig.10 and

Fig.11 list the detailed error distributions. It can be found that the maximum errors locate at the edge of the reactor core. It is because the powers at these locations are small.

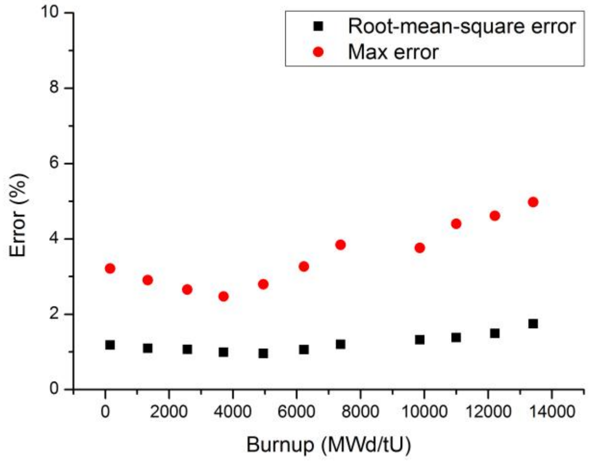

Fig. 9 Root-mean-square errors and maximum errors of radial average power distribution for different burnup levels

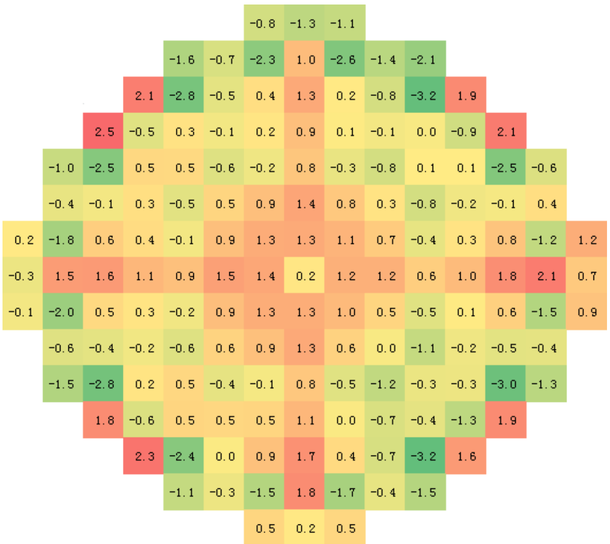

Fig. 10 Assembly radial average power distribution relative errors (%) (150MWd/tU)

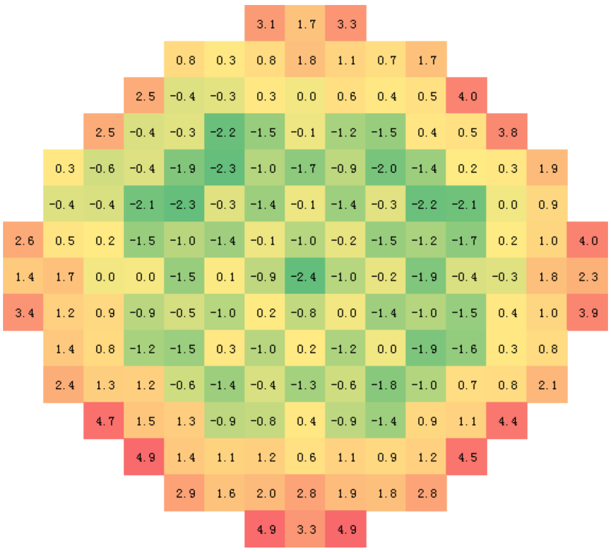

Fig. 11 Assembly radial average power distribution relative errors (%) (13414MWd/tU)

3.3 Discussion about detector failure

To validate NECP-ONION's ability for detector failure, detectors in several channels were removed. Table 6 listed the considered cases and the errors in these cases compared with the commercial code results afforded by the plant with all detectors work. These cases were ranged by the failure detectors' locations in the reactor core. Failure detectors in Case 1 to Case 5 are the detectors in circle 1 to 5 respectively in Fig.12.

Table 6 Detector failure cases considered

Case	Failure detector	Number of failure detectors	RMS error	Max error
1	G07 G09 J07	3	1.12	3.14
2	F06 F08 F09 H06 J10	5	1.08	3.25
3	D07 E05 E11 F11 H04 H11 J05 J12 L05 L06 L08 L09 L11	13	1.09	3.37
4	C08 D05 D10 F04 F13 H03 H13 J03 L04 N07 N08 N10	12	1.24	3.69
5	A09 B05 B07 B08 B10 C12 D03 F02 H01 J15 L14 M03 N05 N12 R08	15	1.11	3.30
6	G07 G09 J07 F06 F08 F09 H06 J10	8	1.29	2.79
7	G07 G09 J07 D07 E05 E11 F11 H04 H11 J05 J12 L05 L06 L08 L09 L11	16	1.10	3.31
8	F06 F08 F09 H06 J10 D07 E05 E11 F11 H04 H11 J05 J12 L05 L06 L08 L09 L11	18	1.07	3.36
9	G07 G09 J07 F06 F08 F09 H06 J10 D07 E05 E11 F11 H04 H11 J05 J12 L05 L06 L08 L09 L11	21	2.54	5.49

From Table 6, cases from 1 to 8 have well monitored accuracy. But when the number of failure detectors is larger than 19 or the failure rate is larger than 40%, the monitored accuracy becomes unacceptable. Therefore, one can say that NECP-ONION can perform well even if only 60% detectors work. It has to be noticed that the detectors failure was performed by removing all the detectors in these channels. But actually, it is almost impossible that all detectors that locate in the same assembly fail. And the number of failed detector is also usually less than 40%.

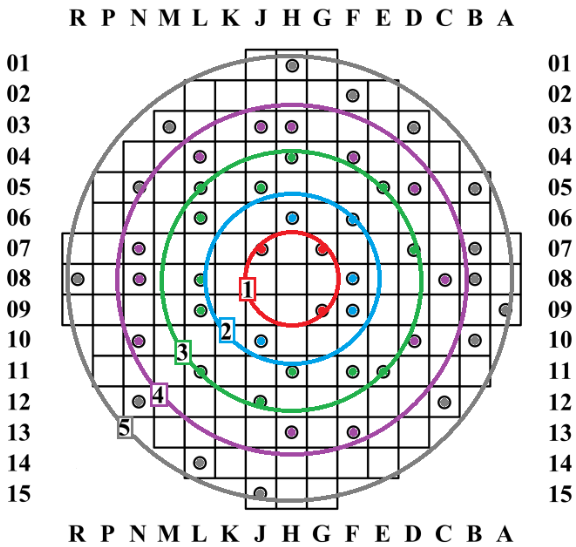

Fig. 12 Failure detector locations

4. CONCLUSION AND FUTURE WORK

In this paper, studies were concentrated on on-line monitoring method, V&V and detector failure. A 3D power distribution on-line monitoring system for PWR named NECP-ONION has been developed based on harmonics expansion method and NLSANM. A typical PWR reactor core was used to validate and verify the accuracy and ability of NECP-ONION system.

Two methods for harmonics calculation were performed. It has been found that NLSANM runs about 100 times faster than FMFDM in harmonics calculation. NECP-ONION provides high accuracy compared with nuclear plant monitored data. From BOC to EOC, the root-mean-square errors and maximum errors of radial average power distributions are all less than 1.8% and 5% respectively when compared with nuclear plant monitored data. A small number detectors failed has little effect about the on-line monitoring system. The largest tolerable number of failed detectors is less than 40%.

As mentioned in Section 1, there are two assumptions about detector signal treatment and core state parameters in this paper. And these parts would be considered and added in the NECP-ONION in the future.

ACKNOWLEDGEMENTS

This work is supported by the National Science Foundation of China (approved number: 91126005 and 91226106).

REFERENCES

Luo, Z.P., et al. 2000, Chinese Journal of Nuclear Science and Engineering, Vol. 20, 32-38. (In Chinese)

Wang, C.H., et al. 2011, Nuclear Engineering and Design, Vol. 241(2011), 3042-3050.

Terney, W., et al. 1983, Transactions of the American Nuclear Society, Vol. 44, 542-543.

Li, F. 1997, Progress in Nuclear Energy, Vol. 31(4), 369-372.

Beard, C. and Morita, T., 1988, Transactions of the American Nuclear Society, Vol. 56, 518-519.

Chan, P.S.W. and Mamourian, M., 1990, Third International Conference on Simulation Methods in Nuclear Engineering, Montreal, Canada.

Lee, K.B., 2003, Nuclear Science and Engineering, Vol. 143, 268-280.

Bonalumi, R.A. and Kherani, N.P., 1985, Nuclear Science and Engineering, Vol. 90, 47-59.

Hong, I.S., 2004, Flux mapping methods for CANDU reactors based on least squares principle and error shape synthesis, Ph.D. thesis, Seoul, Korea.

Beard, C. and Morita, T., 1988, Proceeding of International Reactor Physics Conference, Jackson Hole.

Jeong, C.J. and Cho, N.Z., 2000, Journal of Nuclear Science and Technology, Vol. 37(9), 758-768.

Hong, I.S. and Kim, C.H., 2005, Nuclear Science and Engineering, Vol. 150, 299-309.

Li, F., 1996, Harmonics Synthesis Method for Core Flux Distribution Reconstruction and its Application to Diagnosis, Ph.D. thesis, Beijing, China. (In Chinese)

Belchior, A., et al. 1991, Transactions of the American Nuclear Society, Vol. 63, 420.

Warsa, J.S., et al. 2004, Nuclear Science and Engineering, Vol. 147(1), 218-248.

Verdu, G., et al. 1999, Annals of Nuclear Energy, Vol. 26(7), 579-593.

Wright, T.G. and Trefethen, L.N., 2002, Journal on Scientific Computing, 23(2), 591-605.

Liao, C.K., 2002, Study on Numerical Method for Three Dimensional Nodal Space-Time Neutron Kinetic Equations and Coupled Neutronic/Thermal-Hydraulic Core Transient Analysis, Ph.D. thesis, Xi'an, China. (In Chinese)

Mishra, S., et al. 2012, Nuclear Science and Engineering, Vol. 170, 280-289.